INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN FLUIDS
*Int. J. Numer. Meth. Fluids* 2006; **50**:491–509
Published online 1 September 2005 in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/fld.1075

# Control of transient flow in irrigation canals using Lyapunov fuzzy filter-based Gaussian regulator

Ömer Faruk Durdu*,†

*Faculty of Agriculture, Irrigation and Drainage Department, Adnan Menderes University, 09100 Aydin, Turkey*

## SUMMARY

An optimal fuzzy filter was applied to solve the state estimation problem of the controlled irrigation canals. Using linearized finite-difference model of the open-channel flow, a canal operation problem was formulated as an optimal control problem and an algorithm for gate openings in the presence of unknown external disturbances was derived. A fuzzy filter was designed to estimate the state variables at the intermediate nodes based upon measured values of depth at the points in the canal. A Lyapunov function was utilized as a performance index to formulate the fuzzy interference rules of the optimal fuzzy filter. A linear quadratic Gaussian (LQG) optimal controller for a multi-pool irrigation canal was considered as an example. The state estimation problem in the controller was simulated using two techniques: Kalman estimator and the proposed fuzzy filter. The performance of the fuzzy state estimator designed using the Lyapunov fuzzy technique was compared with the results obtained using the Kalman estimator technique. The obvious advantages of the fuzzy filter were the lower computational costs and ease of implementation. The results of this study demonstrated that proposed Lyapunov-type fuzzy filter provides both good stability and simplicity in the control of irrigation canals more than a Kalman filter.


KEY WORDS: open-channel flow; Lyapunov fuzzy filter; Kalman filter; canal automation

## INTRODUCTION

The goal of modern irrigation system technology is to improve the system's ability to respond to user demands. The assumption behind such an effort is that water delivery in the amounts, at times, and at the rates required by users will lead to better water management on-farm. This calls for a demand delivery of water to the farmers in the command area of an irrigation project. Demand delivery offers the maximum flexibility and convenience to the water user. Receiving water on demand also has economic value to the water user because delays or quantity restrictions are not involved. Although a demand delivery schedule offers flexibility

*Correspondence to: Ömer Faruk Durdu, Faculty of Agriculture, Irrigation and Drainage Department, Adnan Menderes University, 09100 Aydin, Turkey.
†E-mail: odurdu@adu.edu.tr

*Received 17 February 2005*
*Revised 12 July 2005*
*Accepted 13 July 2005*

from the farmer's point of view, the system operation becomes difficult and costly because of the unknown demands or variations in demands (changes in withdrawals) [1]. To meet the unknown demands and to achieve a balanced operation in the canal system, a constant water level control at downstream end of a given pool is employed. In a demand delivery schedule, under constant-level control, since the demands (disturbances or flow rate changes) are not known in advance, the effect of the random disturbances on the system variables must be measured and used in the feedback loop to control the system [1]. The variation in the depths of flow is used in the closed loop (feedback loop). During the few decades, several control algorithms have been developed to derive the relationship between the deviations in the system variables (flow depth and flow rate) and the change in gate opening (gate-control algorithm). In the past, the concepts of optimal control theory have been applied for deriving closed-loop-control algorithms for real-time control of irrigation canals [2–7]. However, when lumped parameter models are used to derive control algorithms for irrigation canals, the number of state variables (flow depths and flow rates) that must be used in the feedback loop is large [8]. Consequently, it is costly to measure flow depths and flow rates at several points in a multi-pool irrigation canal. Therefore, to minimize the cost of implementing feedback-control algorithms, the number of measurements per pool must be kept to an absolute minimum. Since one or two flow depths per pool are normally measured in practice, it is preferable (and possible) to estimate values for the state variables that are not measured. This is done by using an observer or estimator. An observer is a mathematical model of the system that estimates the values for the state variables that are not measured based upon the measured values for one or two more state variables in the pool [1]. The Kalman filtering technique is applied to design an observer for a multi-pool irrigation canal [1, 9]. The Kalman filter is well known for its use in optimal estimation, and is especially suitable for the system with disturbances and measurement errors. A significant difficulty in designing Kalman filters is how to effectively determine the process noise covariance matrix ($Qx$) and the measurement noise covariance matrix ($R$) [10]. These matrices are not usually known precisely, or even in a time varying manner. Therefore, the use of fuzzy estimator theory to deal with the state estimation problem has become of interest to overcome the determination of $Qx$ and $R$ matrices [11–15]. Many studies have been carried out into calibrating the covariance matrices $Qx$ and $R$ by means of fuzzy rules [11]. In general, the design strategies for the fuzzy decision rules are based on heuristic approaches or on the experiences of human experts. The objective of this paper is to apply an optimal fuzzy filter technique in the control of a multi-pool irrigation canal and to compare the performance of the regulators designed using optimal state feedback (target-loop function), Kalman filtering and Lyapunov fuzzy filter techniques.

## BASIC HYDRODYNAMICS OF OPEN-CHANNEL FLOW

The one-dimensional equations (the St Venant equations) for gradually varied, unsteady flow in a prismatic channel are

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_1 \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial (Q^2/A)}{\partial x} + gA\left(\frac{\partial y}{\partial x} - S_0 + S_f\right) = 0 \tag{2}$$

in which $Q$ is the flow rate ($m^3/s$), $A$ the wetted area ($m^2$), $q_l$ the lateral flow ($m^2/s$), $y$ the water depth (m), $t$ the time (s), $x$ the longitudinal direction of channel (m), $g$ the gravitational acceleration ($m^2/s$), $S_0$ the canal bottom slope (m/m), $n$ the roughness coefficient ($s/m^{1/3}$), and $S_f$ the friction slope (m/m) and is defined as

$$S_f = \frac{Q|Q|}{K^2} \tag{3}$$

in which $K$ is the hydraulic conveyance of canal $= AR^{2/3}/n$, $R$ the hydraulic radius (m). In Equations (1) and (2), the spatial derivatives were replaced by finite-difference approximations, by dividing the pool into few segments ($N =$ number of nodes). A central-difference scheme was used for the interior nodes ($1 < j < N$), and a forward difference and a backward difference were applied to the first and the last nodes, respectively [5]. Both forward and backward finite-difference approximations (the neglected terms are of the first order of $\Delta x$) are referred to as first-order accurate. Central finite-difference approximation (the neglected terms is of the order of $(\Delta x)^2$) is referred to as second-order accurate [16]. The central finite-difference approximation is more accurate than the forward or backward finite-difference approximations. The water levels or the volumes of water stored in the canal pools are regulated using a series of spatially distributed gates (control elements). Hence, open irrigation canals are modelled as distributed control systems. To solve Equations (1) and (2), the boundary conditions were expressed in terms of the continuity equation:

$$Q_{i-1,N} = Q_{i,1} = Q_{gi} \tag{4}$$

and gate discharge equation [5]

$$Q_{gi} = C_{di} b_i u_i (2g(Z_{i-1,N} - Z_{i,1}))^{1/2} \tag{5}$$

in which, $Q_{i-1,N}$ is the flow rate through downstream gate (or node $N$) of pool $i-1$ ($m^3/s$), $Q_{gi}$ the flow rate through upstream gate of pool $i$ ($m^3/s$), $Q_{i,1}$ the flow rate through upstream gate (or node 1) of pool $i$ ($m^3/s$), $C_{di}$ the discharge coefficient of gate $i$, $b_i$ the width of gate $i$ (m), $u_i$ the opening of gate $i$ (m), $Z_{i-1,N}$ the water surface elevation at node $N$ of pool $i-1$ (m), $Z_{i,1}$ the water surface elevation at node 1 of pool $i$ (m) and $i$ the pool index ($i = 0$ refers to the upstream constant-level reservoir). In the algorithm, the turnouts were assumed to be scattered throughout the pool length (Figure 1). The withdrawals were related to $q_l$ of Equation (1) as follows [5]:

$$q_l = \frac{\sum_{n=1}^{O_{i,j}} q_{i,n}}{s} \tag{6}$$

where $s = \Delta x$ in the case of backward/forward difference scheme, and $s = 2\Delta x$ in a central-difference scheme; $\Delta x$ is the distance between two nodes (m), $q_{i,n}$ the withdrawal rate from outlet $n$ of pool $i$ ($m^3/s$) and $O_{i,j}$ the number of outlets represented around node $j$ of pool $i$. Linear optimal control theory is well developed and is easier to apply than nonlinear control theory [17]. The linearized model was derived based upon the initial steady-state condition. Equations (1) and (2) describe the conservation of mass and momentum in terms of the partial derivatives of dependent variables: flow depth and velocity. For practical applications, the values of these variables instead of the values of their derivatives must be known. Because of the presence of nonlinear terms, a closed-form solution of these equations is not available,

 

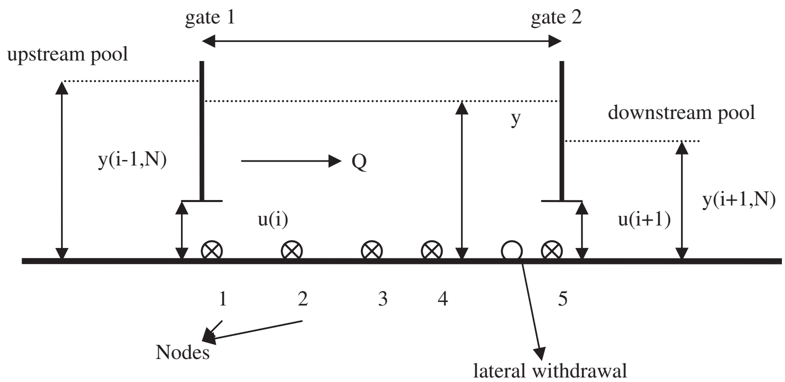


Figure 1. Schematic diagram of a single pool.

except for very simplified cases [18]. The stability of a numerical solution of the equations can be investigated by studying whether an error grows or decays as the solution progress in a marching (step-by-step) procedure. Rigorous procedures are not presently available to determine the stability of nonlinear equations. However, by linearizing the nonlinear terms of the St Venant equations, stability can be studied. If the nonlinearities are not strong, then the criteria developed for the linear equations may be assumed to be valid for the nonlinear equations as well. Although, the employed finite-difference scheme for the solution of the equations is plausible, at each step of the solution an error is introduced, due to the approximation of the partial differentials by finite quotients. This error has been labelled the truncation or discretization error, and depends on the magnitude of the time and space intervals selected [19]. Although at a given time step the magnitude of the discretization error may appear small in comparison with the calculated quantities, a practical calculation may involve thousands of consecutive time steps, so that it is absolutely essential to verify that these errors remain small in magnitude with respect to the true solution. In order to describe the behaviour of the numerical solution it is usual to consider the properties of stability, which must be satisfied by the numerical solution to be employed in practical computation [19]. Therefore, Equations (1) and (2) were linearized numerically about an average operation condition. Using the Taylor series around the equilibrium point and truncating terms higher than the first order, the deviation variables were defined as follows [5]:

$$\delta u_i = u_i - u_i^0 \tag{7}$$

$$\delta Z_{i,j} = Z_{i,j} - Z_{i,j}^0 \tag{8}$$

$$\delta q_{i,n} = q_{i,n} - q_{i,n}^0 \tag{9}$$

$$\delta Q_{i,j} = Q_{i,j} - Q_{i,j}^0 \tag{10}$$

in which $\delta Q_{i,j}$ is the variation in flow rate at node $j$ of pool $i$ ($m^3/s$), $\delta Z_{i,j}$ the variation in water surface elevation at node $j$ of pool $i$ (m), $\delta u_i$ the variation in upstream gate opening of pool $i$ (m), $\delta q_{i,n}$ the variation in water withdrawal rate from outlet $n$ of pool $i$ [($m^3/s$)/m]. The truncation and other errors are introduced at any step in the computations one defines by stability the property of the numerical solution that these errors remain bounded, that they do not amplify without limit as the time interval tends to zero. Substitution of Equations (7)–(10) into Equations (1) and (2) and application of the finite-difference implicit technique result in a set of linear, ordinary differential equations for the canal with control gates and turnouts [3, 20]

$$A_{11}\delta Q_j^+ + A_{12}\delta z_j^+ + A_{13}\delta Q_{j+1}^+ + A_{14}\delta z_{j+1}^+ = A'_{11}\delta Q_j + A'_{12}\delta z_j + A'_{13}\delta Q_{j+1} + A'_{14}\delta z_{j+1} + C_1 \tag{11}$$

$$A_{21}\delta Q_j^+ + A_{22}\delta z_j^+ + A_{23}\delta Q_{j+1}^+ + A_{24}\delta z_{j+1}^+ = A'_{21}\delta Q_j + A'_{22}\delta z_j + A'_{23}\delta Q_{j+1} + A'_{24}\delta z_{j+1} + C_2 \tag{12}$$

where $\delta Q_j^+$ and $\delta z_j^+$ are the discharge and water-level increments from time level $t+1$ at node $j$, $\delta Q_j$ and $\delta z_j$ the discharge and water-level increments from time level $t$ at node $j$, and $A_{11}, A'_{21}, \ldots\ A_{12}, A_{22}$ are the coefficients of the continuity and momentum equations, respectively, computed with known values at time level $t$. Similar equations are derived for channel segments that contain a gate structure, a weir or some other type of hydraulic structure. From the above equations, the state of system equation at any sampling interval $k$ can be written, in a compact form, as follows [10]:

$$A_\mathrm{L}\delta x(k+1) = A_\mathrm{R}\delta x(k) + B\delta u(k) + C\delta q(k) \tag{13}$$

where $A$ is the $n\times n$ system feedback matrix, $B$ the $l\times m$ control distribution matrix, $C$ the $p\times l$ disturbance matrix, $\delta x(k)$ the $l\times 1$ state vector, $\delta u(k)$ the $m\times 1$ control vector, $\Delta\delta q$ the variation in demands (or disturbances) at the turnouts (m$^2$/s), $l$ the number of dependent (state) variables in the system, $m$ the number of controls (gates) in the canal, $p$ the number of outlets in the canal, and $k$ the time increment (s). The elements of the matrices $A, B$, and $C$ depend upon the initial condition [7]. The dimensions of the control distribution matrix, $B$, depend on the number of state variables and the number of gates in the canal. The dimensions of the disturbance matrix, $C$, depend on the number of disturbances acting on the canal system and the number of dependent state variables. Equation (13) can be written in a state-variable form along with the output equations as follows [1]:

$$\delta x(k+1) = \Phi\delta x(k) + \Gamma\delta u(k) + \Psi\delta q(k) \tag{14}$$

$$\delta y(k) = H\delta x(k) \tag{15}$$

where $\Phi = (A_\mathrm{L})^{-1} * A_\mathrm{R}$, $\Gamma = (A_\mathrm{L})^{-1} * B$, and $\Psi = (A_\mathrm{L})^{-1} * C$, $\delta y(k) = r\times 1$ vector of output (measured variables), $H = r\times l$ output matrix, and $r =$ number of outputs. The elements of the matrices $\Phi, \Gamma$, and $\Psi$ depend upon the canal parameters, the sampling interval, and the assumed average operating condition of the canal. In Equation (14), the vector of state variables is defined as follows [21]:

$$\delta x = (\delta Q_{i,1}, \delta Z_{i,2}, \delta Q_{i,2}, \ldots, \delta Z_{i,N-1}, \delta Q_{i,N-1}, \delta Q_{i,N}) \tag{16}$$

 

QUADRATIC GAUSSIAN (LQG) REGULATOR

The LQG theory provides an integrated knowledge base for the development of a flexible controller. The LQG controller integrates the states estimation and the controller design (Figure 2) into a single body of knowledge (Figure 3). An LQG controller consists of an optimal state feedback (LQR) and an optimal state estimator. An optimal LQG controller based upon a linear system, a quadratic objective function and an assumption of white noise that has a normal, or Gaussian, probability distribution. In this study, an LQR controller was first designed and simulated based on given example canal. Then a Kalman filter subroutine was inserted into the controller algorithm and the algorithm was simulated to observe the effects of the estimated state variables on the performance of the system. Later, the Kalman state estimator was replaced by a Lyapunov fuzzy estimator subroutine. The control algorithm was simulated again for the canal to observe the performance of the fuzzy estimator in comparison to the Kalman filter.

### *Design of optimal state feedback*

Linear quadratic regulator (LQR) control problem as an optimization problem in which the cost function, $J$, to be minimized is given as follows [10]:

$$J = \sum_{i=1}^{K_\infty} [\delta x^{\mathrm{T}}(k) Q x_{l\times l} \delta x(k) + \delta u^{\mathrm{T}}(k) R_{m\times m} \delta u(k)] \tag{17}$$

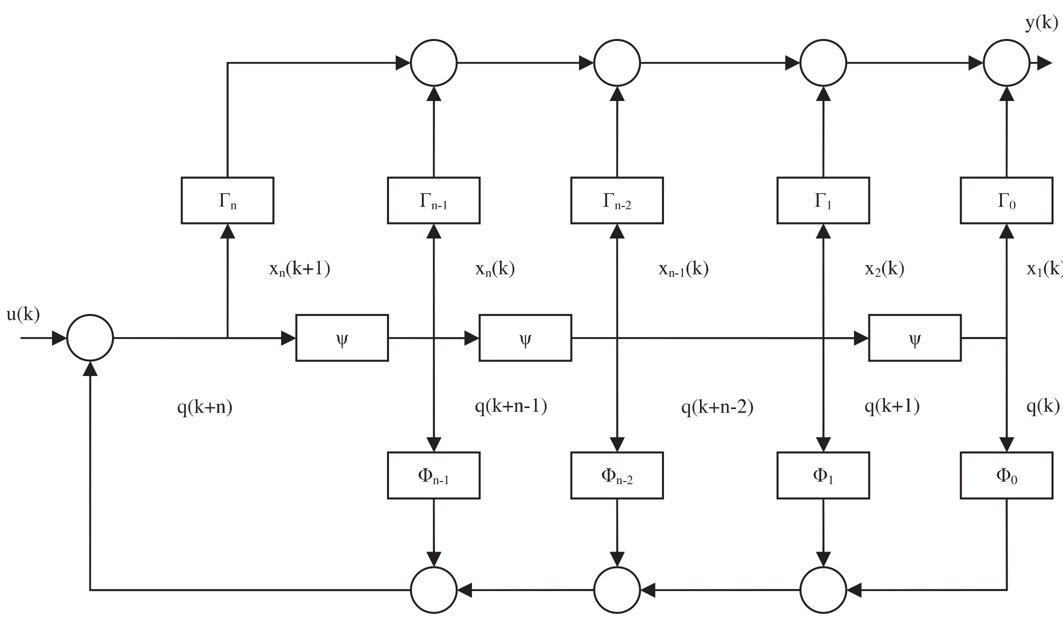


Figure 2. A multivariable feedback control system scheme.

 

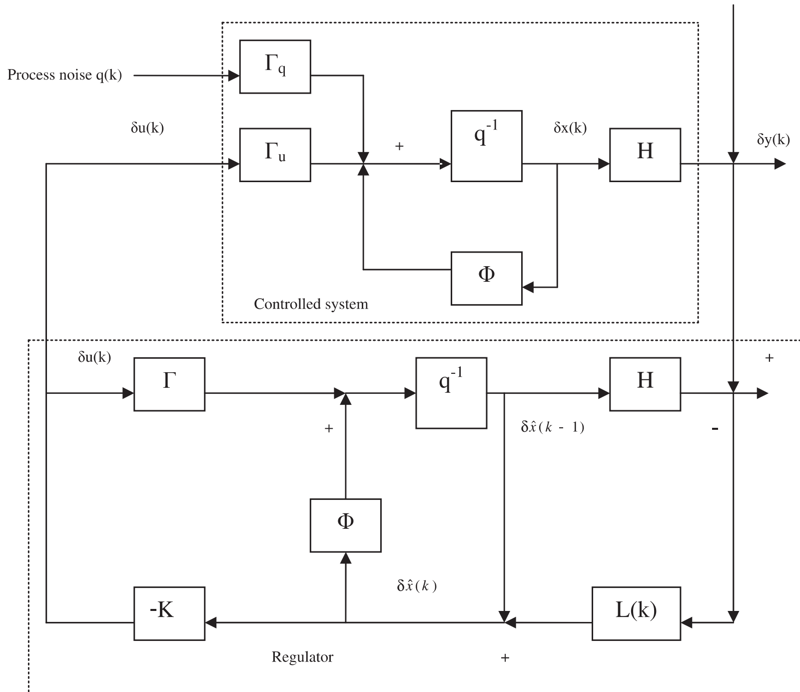


Figure 3. Linear quadratic Gaussian (LQG) controller with a state estimator.

subject to the constraint that:

$$-\delta x(k+1) + \Phi\delta x(k) + \Gamma\delta u(k) = 0 \quad k = 0, \ldots, K_{\infty} \tag{18}$$

where $K_{\infty}$ is the number of sampling intervals considered to derive the steady-state controller, $Qx_{l\times l}$ the state cost weighting matrix, and $R_{m\times m}$ the control cost weighting matrix (well-known steady-state solution of a Riccati equation). The matrices $Qx$ and $R$ are symmetric, and to satisfy the non-negative definite condition, they are usually selected to be diagonal with all diagonal elements positive or zero. Kailath [22] defined that the choice of the elements of these matrices is more of an art than a science. Flow transition in the canal is defined by specifying a rough estimate of the time, $t$, and deviations in velocity, $\Delta V$. As $t$ is increased, a higher penalty is applied to depth deviations and gate velocities. The effect of a larger $t$ specification will be less abrupt responses to changes in the disturbances [18]. The maximum deviations in gate openings ($\delta u$) are just the difference between the initial and final steady gate position. On the diagonal elements of $Qx$ and $R$ are nonzero and their values are determined from the flow transition and the corresponding wave equation response [18]. Conditions for existence and uniqueness of the optimal solution are always met when diagonal penalization is used. The first term, $\delta x^{\mathrm{T}}(k)Qx_{l\times l}\delta x(k)$, in Equation (17) represents the penalty on the deviation of the state variables from the average operating (or target) condition. The second term, $\delta u^{\mathrm{T}}(k)R_{m\times m}\delta u(k)$, called as the cost of control, contains quadratic functions of the elements of gate deviation matrix ($\delta u$). This term is included in an attempt to limit the magnitude of the control signal $\delta u(k)$. Unless a cost is imposed for use of control, the design that emerges is liable to generate control signals that cannot be achieved by the actuator. In this case the saturation of the control signal will occur resulting in a system behaviour that is different from the closed-loop system behaviour that was predicted assuming that saturation will not occur [7]. Therefore, the control signal weighting matrix elements are selected to be large enough to avoid saturation of the control signal under normal operating conditions. Equations (17) and (18) constitute a constrained-minimization problem that can be solved using the method of Lagrange multipliers [7]. This produces a set of coupled difference equations which must be solved recursively backwards in time. In the optimal steady-state case, the solution for change in gate opening, $\delta u(k)$, is of the same form as

$$\delta u(k) = -K\delta x(k) \tag{19}$$

where $K$ is given by

$$K = [R + \Gamma^{T} S \Gamma]^{-1} \Gamma^{T} S \Phi \tag{20}$$

$S$ is a solution of the discrete algebraic Riccati equation (DARE)

$$\Phi^{T} S \Phi - \Phi^{T} S \Gamma [R + \Gamma^{T} S \Gamma]^{-1} \Gamma^{T} S \Phi + Q_x = S \tag{21}$$

where $R = R^{T} > 0$ and $Qx = Qx^{T} = H^{T} H \geqslant 0$. The control law defined by Equation (19) brings an initially disturbed system to an equilibrium condition in the absence of any external disturbances acting on the system [7]. In the presence of these external disturbances, the system cannot be returned to the equilibrium condition using the Equation (19). An integral control, in which the cumulative (or integrated) deviation of a selected output variable is used in the feedback control loop, is required to return the system to the equilibrium condition in the presence of external disturbances [17]. Integral control is achieved by appending additional variables of the following form to the system dynamic equation:

$$\delta x_{I}(k+1) = D\delta x(k) + \delta x_{I}(k) \tag{22}$$

in which $\delta x_{I}$ is the integral state variables and $D$ the integral feedback matrix. This produces a new control law [7]:

$$\delta u(k) = -K\delta x(k) - K_{I}\delta x_{I}(k) \tag{23}$$

The first term in Equation (23) accounts for initial disturbances, whereas the second term accounts for external disturbances. Equation (23) predicts the desired gate openings as function of the measured deviations in the values of the state variables [5]. In this paper, the water surface elevation and flow rate were considered as state variables. Given initial conditions [$\delta x(0)$], $\delta u$, and $\delta q$, Equation (17) can be solved for variations in flow depth and flow rate as a function of time. If the system is really at equilibrium [i.e. $\delta x(0) = 0$ at time $t = 0$] and there is no change in the lateral withdrawal rates (disturbances), the system would continue to be at equilibrium forever; then, there is no need for any control action. Conversely, in the presence of disturbances (known or random), the system would deviate from the equilibrium condition [4]. The actual condition of the system may be either above or below the equilibrium condition, depending upon the sign and magnitude of the disturbances. If the system deviates significantly from the equilibrium condition, the discharge rates into the laterals will be different (either more or less) than the desired values. But in canal operations, the main objective is to keep these deviations to a minimum so that a nearly constant rate of discharge is maintained through the turnouts.

*Design of Kalman filter*

The LQG theory provides an integrated knowledge base for the development of a flexible controller. Since it is expensive to measure all the state variables (flow rates and flow depths) in a canal system, the number of measurements per pool must be kept to an absolute minimum. Usually, the flow depths at the upstream and downstream ends of each pool are measured. The relationship between the state variables and the measured (or output) variables is [7]

$$\delta y(k) = H\delta x(k) + \eta(k) \tag{24}$$

where $\eta(k)$ is the measurement error inputs. For steady-state Kalman filter, the observer gain matrix, $L$, is calculated as follows:

$$L = PH^{\mathrm{T}}[HPH^{\mathrm{T}} + RC]^{-1} \tag{25}$$

where $P$ is the covariance of estimation uncertainty:

$$\Phi^{\mathrm{T}}P\Phi - \Phi PH^{\mathrm{T}}[RC + H^{\mathrm{T}}PH]^{-1}HP\Phi^{\mathrm{T}} + Q_{\mathrm{esti}} \tag{26}$$

where $RC = RC^{\mathrm{T}} > 0$ is a tolerance values for the $RC$ covariance matrix which is an identity matrix and $Q_{\mathrm{esti}} = Q_{\mathrm{esti}}^{\mathrm{T}} \geqslant 0$ is a diagonal matrix. The disturbances $\delta q(k)$ and $\eta(k)$, in Equations (14) and (24), are assumed to be zero mean Gaussian white noise sequences with symmetric positive definite covariance matrices $Q_{\mathrm{esti}}$ and $RC$, respectively. Furthermore, sequences $\delta q(k)$ and $\eta(k)$ are assumed to be statistically independent. The system dynamic equation is used to predict the state and estimation error covariance as follows. Time-update equations [23]:

$$P_{-}(k+1) = \Phi P(k)\Phi^{\mathrm{T}} + \Psi Q_{\mathrm{esti}}\Psi^{\mathrm{T}} \tag{27}$$

$$\delta\hat{x}_{-}(k+1) = \Phi\delta\hat{x}(k) + \Gamma\delta u(k) \tag{28}$$

in which $\delta\hat{x}(k)$ is the estimated values of the state variables. As soon as measured values for the output variables $\delta y(k)$ are available, the time-update values are corrected using the measurement-update equations as follows. Measurement-update equations [23]:

$$L(k+1) = P_{-}(k+1)H^{\mathrm{T}}[HP_{-}(k+1)H^{\mathrm{T}} + RC]^{-1} \tag{29}$$

$$P(k+1) = [I - L(k+1)H]P_{-}(k+1) \tag{30}$$

$$\delta\hat{x}(k+1) = \delta\hat{x}_{-}(k+1) + L(k+1)[\delta y(k+1) - H\delta\hat{x}_{-}(k+1)] \tag{31}$$

If the initial conditions and the inputs (control inputs and the disturbances) are known without error, the system dynamic equation, Equation (2), can be used to estimate the state variables that are not measured [24]. Since part of the disturbances are random and usually are not measured, the canal parameters are not known very accurately, the estimated values of the state variables would diverge from the actual values. This divergence can be minimized by utilizing the difference between measured output and the estimated output (error signal), and by constantly correcting the system model with the error signal [1]. Therefore, the modified state equations are given as

$$\delta\hat{x}(k+1)=\Phi\delta\hat{x}(k)+\Gamma\delta u(k)+L[\delta y(k)+H\delta\hat{x}(k)] \tag{32}$$

*Design of Lyapunov-type fuzzy filter*

Designing an estimator for the multi-pool irrigation system is assuming a known control input (variations in gate opening), $\delta u(k)$, a measured output, $\delta y(k)$, measurement error noise, $\eta(k)$, and disturbance noise, $\delta q(k)$. The estimator is designed to provide an optimal estimate of the state vector, $\delta x(k)$. Combination of the optimal state feedback and state estimator generates the input vector, $\delta u(k)$, based upon the estimated state vector, $\delta\hat{x}(k)$, rather than the actual state vector, $\delta x(k)$, and the measured output vector, $\delta y(k)$. The application of fuzzy theory to the state estimation problem is a rising topic [11]. The Lyapunov theorem is one of the most useful methods when one deals with stability problem for the conventional control systems. And it is quite simple and easy to be implemented. It simply depends upon setting up a positive definite function $V(k)$ and then verifying $V(k)$'s derivative, $\bar{V}(k)$, is negative. In this study, the Lyapunov function and the derived sensitivity function were used as performance indices to organize the fuzzy interference rules. The Lyapunov-function-based fuzzy filter for the linear systems is organized as follows [25]:

$$\delta\hat{x}_{\mathrm{LF}}(k/k-1)=\Phi\delta\hat{x}(k-1/k-1)+\Gamma\delta u(k-1) \tag{33}$$

$$\delta\hat{x}_{\mathrm{LF}}(k/k)=\delta\hat{x}_{\mathrm{LF}}(k/k-1)+L(k)[\delta y(k)-H\delta\hat{x}_{\mathrm{LF}}(k/k-1)] \tag{34}$$

where $\delta\hat{x}_{\mathrm{LF}}(k/k-1)$ and $\delta\hat{x}_{\mathrm{LF}}(k/k)$ are the predicted state and the updated state, respectively, and $L(k)$ is the fuzzy correction gain matrix that is designed not only to guarantee convergence but also to make the estimated state $\delta\hat{x}_{\mathrm{LF}}(k/k)$ approach the true state $\delta x(k)$ as soon as possible [11]. Consider the Lyapunov function

$$V(k)=e(k)^{\mathrm{T}}\cdot e(k) \tag{35}$$

where $e(k)=H[\delta x(k)-\delta\hat{x}_{\mathrm{LF}}(k/k)]$ is the state estimation error vector. The main objective for the proposed fuzzy filter is to determine $L(k)$ such that the Lyapunov difference is guaranteed to be negative, i.e. $\Delta V(k)=V(k)-V(k-1)<0$. Let the sensitivity function, $S(k)$, be defined as [25]

$$S(k)=\frac{\partial V(k)}{\partial L(k)}\cong\frac{V(k)-V(k-1)}{L(k)-L(k-1)}=\frac{\Delta V(k)}{\Delta L(k)} \tag{36}$$

and

$$\begin{aligned}\Delta V(k)&=\sum_i^m\sum_j^n\Delta V_i(k)\cong\sum_i^m\sum_j^n\frac{\partial V_i}{\partial L_{ij}}\Delta L_{ij}=\sum_i^m\sum_j^n\frac{V_i(k)-V_j(k-1)}{L_{ij}(k+1)-L_{ij}(k-1)}\Delta L_{ij}\\&=\sum_i^m\sum_j^n\frac{\Delta V_i(k)}{\Delta L_{ij}(k)}\Delta L_{ij}=\sum_i^m\sum_j^n S_{ij}\Delta L_{ij}\end{aligned} \tag{37}$$

 

where $L_{ij}$ is the *ij*th entry of the fuzzy gain matrix $L$ and $\Delta L_{ij}$ is the degree of variation to be determined. As seen above equations, the fuzzy control scheme is to generate a correct $\Delta L_{ij}$ such that $(\partial V/\partial L_{ij})$. $\Delta L_{ij} < 0$, i.e. the $\Delta V(k)$ will always be negative [11]. The actual *ij*th element of the fuzzy gain matrix is calculated by

$$L_{ij}(k+1) = L_{ij}(k) + \Delta L_{ij}(k+1) \tag{38}$$

In Equation (38), $V(k)$ represents the distance between the estimated state and the actual state. The goal of estimation is to decrease the distance as quickly as possible. Therefore, $V(k)$ is considered as an exponential decaying function. To obtain better performance, the hierarchical construction will be exploited. The desired exponential decaying response is divided into three fuzzy subsets: large, medium and small [25]. Next, the sign of $\Delta V(k)$ indicates whether the state is now diverging from and converging to the actual state $\delta x(k)$. A stronger control action must be taken to drive the divergent states back, and only a medium control command should be required to maintain the movement of the estimated states towards the actual states [11]. When $V(k)$ is small and $\Delta V(k)$ is negative, a smaller control amount is sufficient for obtaining an estimation. In this study the fuzzy rules for the fuzzy filter are expressed as

If $\Delta V(k)$ is positive and medium and $S(k)$ is negative and large, then $\Delta L(k+1)$ is positive and small.
If $\Delta V(k)$ is positive and zero and $S(k)$ is negative and medium, then $\Delta L(k+1)$ is positive and zero.
If $\Delta V(k)$ is negative and large and $S(k)$ is positive and large, then $\Delta L(k+1)$ is negative and zero.
If $\Delta V(k)$ is negative and small and $S(k)$ is positive and zero, then $\Delta L(k+1)$ is negative and medium.

From above fuzzy rules, it is obvious that the sign of $\Delta L_{ij}$ is determined by $S_{ij}$, i.e. opposite to $S_{ij}$. $\Delta V_i(k)$ is used to determine the amount of $\Delta L_{ij}$. Once the equations of the optimal state feedback and the fuzzy observer are obtained, and measured values for one or more state variables for each pool are available, the dynamics of the linear system can be simulated for any arbitrarily selected values of external disturbances. In this study, a multi-pool irrigation canal was considered. The algorithm predicts the flow rate, $Q(x,t)$ and the depth of flow, $y(x,t)$, given the initial boundary conditions [26]. The optimal state feedback and the fuzzy estimator equations were added as subroutines to this program. Given the initial flow rate and the target depth at the downstream end of the each pool, the algorithm computed the backwater curve. Later on, the downstream flow requirement and the withdrawal rate into the lateral were provided as a boundary condition. The model predicted the depths and flow rates at the nodal points for the next time increment. The computed depths at the upstream and downstream ends of the each pool were used with the fuzzy observer under fuzzy rules constraints to estimate the flow depths and flow rates at some selected intermediate nodal points. These estimated values were then used in the optimal state feedback subroutine to compute the change in the upstream gate opening in order to bring the depth at the downstream end of the pool close to the target depth. When the estimated values of the state variables are used in the feedback loop, the controller equation, Equation (19) becomes [7]

$$\delta u(k) = -K\delta\hat{x}(k) - K_I \delta x_I(k) \tag{39}$$

 

Equation (39) computes the desired change in gate opening as a function of the estimated (instead of measured) deviations in the state variables. Based upon this gate opening, the new flow rate into the pool at the upstream end was calculated and used as the boundary condition at the upstream end of the each pool [5]. This process was repeated during the entire simulation period.

## RESULTS AND ANALYSIS

To demonstrate the effectiveness and stability of the Lyapunov-type fuzzy estimator, an LQG regulation problem for a discrete-time multi-pool irrigation canal had been simulated. The data used were as follows: length of canal reach $= 54\,000$ m, number of nodes $= 49$, number of subreaches used $= 6$, $\Delta x = 1500$ m, channel slope $= 0.0002$, side slope $= 1.0$, bottom width $= 5$ m, disturbance along the simulation $= 2.5\,\mathrm{m}^3/\mathrm{s}$, discharge required at the end of the canal $= 5\,\mathrm{m}^3/\mathrm{s}$, target depth at downstream end $= 1.5$ m, gate width $= 5$ m, and gate discharge coefficient $= 0.8$. These data were first used to calculate the steady-state values, which in turn were used to compute the initial gate openings and the elements of the $\Phi, \Gamma, H$ matrices using sampling interval of 30 s. The values of the initial gate openings for gates $1, 2, 3, 4, 5, 6$, and 7 were $1.133, 1.369, 1.168, 0.978, 0.858, 0.636$ and 0.7 m, respectively. After computing steady-state values, the control algorithm formulates an LQG controller with a Kalman filter and a fuzzy estimator, respectively. As a first part of the LQG controller, an optimal state feedback controller (assumed all states variables are available) was designed to regulate the six-pool canal system using a constant-level-control approach. The system response was simulated using the controller in the feedback loop. In the derivation of the feedback gain matrix $K$, the control cost weighting matrix, $R$, of dimensions 6, was set equal to 100, whereas the state cost weighting matrix, $Qx$, was set equal to an identity matrix of dimensions 85. The matrix dimension 85 came from the system dimension. Since the irrigation canal was divided into 49 nodes and each node had a set of two equations, in other words, the dimension of the system should have been equal to 98. But the system had 7 gates and 6 turnouts; therefore, the system matrix dimensions were 85. The cost weighting matrix and the control cost matrix must be symmetric and positive definite (i.e. all eigenvalues of $R$ and $Qx$ must be positive real numbers). *A priori*, we do not quite know what values of $Qx$ and $R$ will produce the desired effect. In the absence of a well-defined procedure for selecting the elements of these matrices, these values are selected based upon trial and error. At first, both $Qx$ and $R$ as identity matrices were selected. By doing so, it was specified that all state variables and control inputs were equally important in the objective function, i.e. it was equally important to bring all the deviations in the state variables (water surface elevations and flow rate) and the deviations in the control inputs to zero while minimizing their overshoots. Note that the existence of a unique, positive definite solution to the algebraic Riccati equation (Equation (21)) is guaranteed if $Qx$ and $R$ are positive semi-definite and positive definite, respectively, and the system is controllable. The analysis started by evaluating the system stability. All the eigenvalues of the system characteristic equation were positive and had values less than one. A good control requires the ability to change the behaviour (controllability) of the system. Controllability is an inherent structural property of a system. The knowledge of controllability is crucial to the subsequent state-variable feedback. Without controllability, not all of the states can be steered in the desired direction by input manipulation. A dynamic system is controllable if a control $\delta u(t)$ exists such that any final state can be attained [4]. The controllability matrix was calculated and the system was found to be controllable. After defining $Qx$ and $R$ matrices, the optimal feedback gain matrix, $K$, was calculated.

Since measurement of all the state variables was expensive, the control algorithm first estimated state variables using a Kalman filter. Next, a fuzzy estimator was employed to estimate the values for the state variables in the algorithm. Kalman filter for the system used the control input $\delta u(k)$, generated by the optimal state feedback, measured water depths $\delta y(k)$ for each pool, the disturbances noise $\delta q(k)$, and measurement noise, $\eta(k)$. In the design of the Kalman filter, *in lieu* of actual field data on withdrawal rates from the turnouts, the random disturbances were assumed to have some prespecified levels of variance. The actual time series of the demands was not used in the design of the filter; only the variance of the time-series was required in the design of the filter. Usually, the sensors used to measure flow depths in open-channel are reasonably accurate to a fraction of a centimeter; therefore, the variance of the measurement error is usually very small [7]. The variances of the disturbances ($Q_{\text{esti}}$) must be estimated from historical records on water withdrawals from the canal outlets. The variances of the disturbances were: $w_1 = 1^2\,\text{m}^2/\text{s m}$, $w_1 = 1.3^2\,\text{m}^2/\text{s m}$, $w_1 = 0.7^2\,\text{m}^2/\text{s m}$, $w_1 = 1.4^2\,\text{m}^2/\text{s m}$ and $w_1 = 1.3^2\,\text{m}^2/\text{s m}$. A value of 0.0005 was used for the variance of the measurement matrix ($RC$), and it was an identity matrix. Using the given initial values, the system response was simulated for 250 time increments or 7500 s. After designing the LQG controller with Kalman filter, the algorithm designed fuzzy estimator based on defined fuzzy rules. To obtain an estimation, an appropriate initial gain matrix, $L(0)$, was chosen. In this study, since it was independent of the initial guess of $P(0)$, the steady-state Kalman gain, $L(0)$, was adopted by using the following equations:

$$\Phi P + P\Phi^{\text{T}} - PH^{\text{T}}R^{-1}HP = \Psi Q_{\text{esti}}\Psi^{\text{T}} \tag{40}$$

$$L(0) = PH^{\text{T}}R^{-1} \tag{41}$$

Once $L(0)$ was computed, the algorithm calculated the dynamic and measurement equations using Equations (14) and (24). Then, time-update equation, $\delta\hat{x}_{\text{LF}}(k/k-1)$, was determined by using Equation (33). Optimal fuzzy estimator determines $L(k)$ such that the Lyapunov difference is guaranteed to be negative, i.e. $\Delta V(k) = V(k) - V(k-1) < 0$. Once update equations are computed, the algorithm calculates the Lyapunov function $V(k)$ using Equation (35). Using Equations (36), (37) and defined fuzzy rules, the variation amount ($\Delta L_{ij}$) of the previous fuzzy gain matrix $L_{ij}$ was determined. Later on, the fuzzy gain matrix, $L$, was computed by using Equation (35). Then, the algorithm calculated the updated estimate state $\delta\hat{x}_{\text{LF}}(k/k)$ using Equation (34). This process was repeated along the simulation until finding guaranteed negative Lyapunov difference.

The analysis was started by evaluating the system stability. All the eigenvalues of the feedback matrix were positive and had values less than one. The system was also found to be both controllable and observable. In the derivation of the control matrix elements, $\Gamma$, it was assumed that both the upstream and downstream gates of each reach could be manipulated to control the system dynamics. The last pool's downstream-end gate position was frozen at the original steady-state value, and only the upstream gates of the given pool were controlled to maintain the system at the equilibrium condition. The effect of variations in the opening of the downstream gate must be taken into account through real-time feedback of the actual depths immediately upstream and downstream of the downstream gate (node $N$). Figure 4 demonstrates the incremental gate openings for each design technique (optimal

 

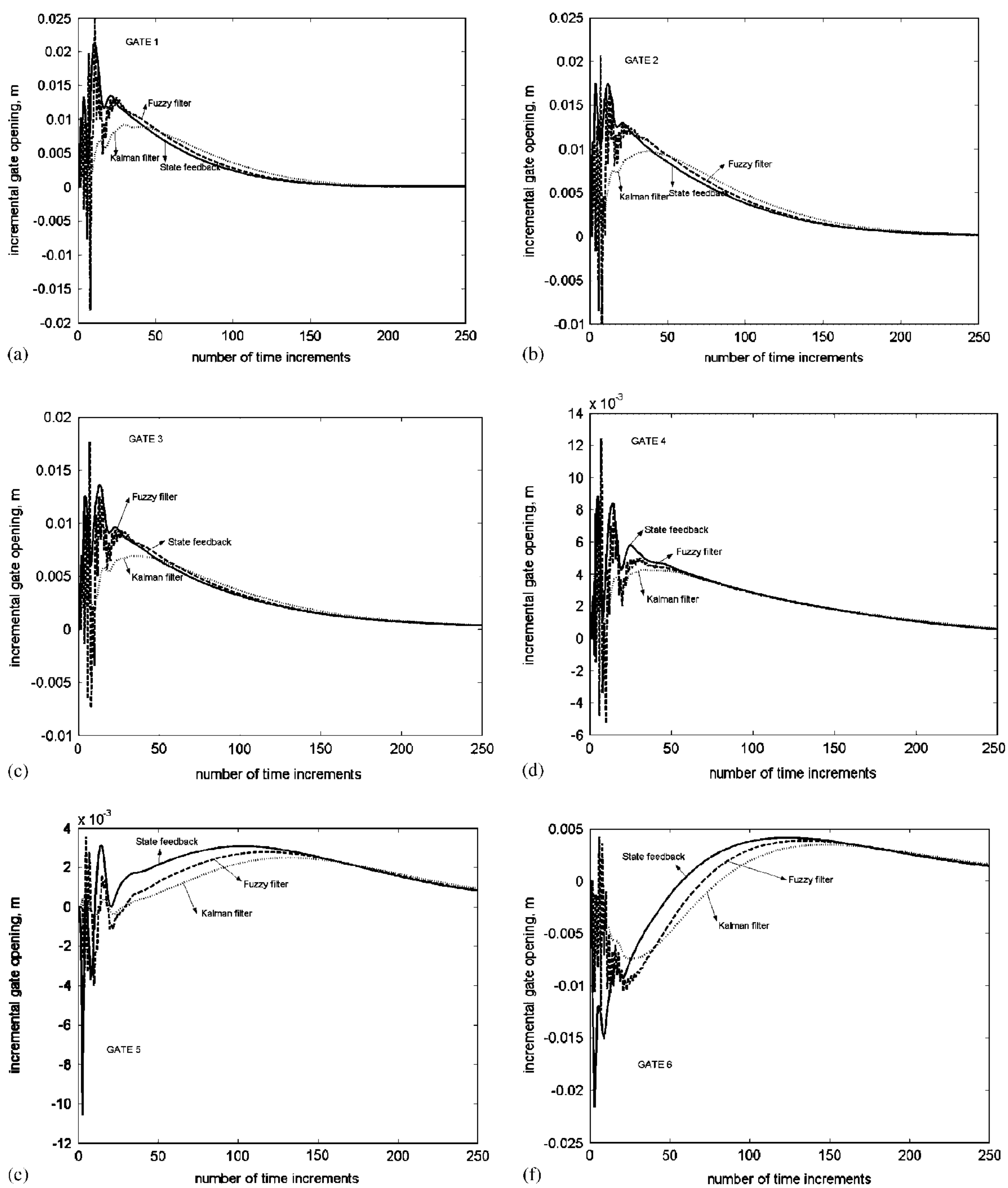


Figure 4. Incremental gate openings for optimal state feedback, Kalman filter and fuzzy filter.

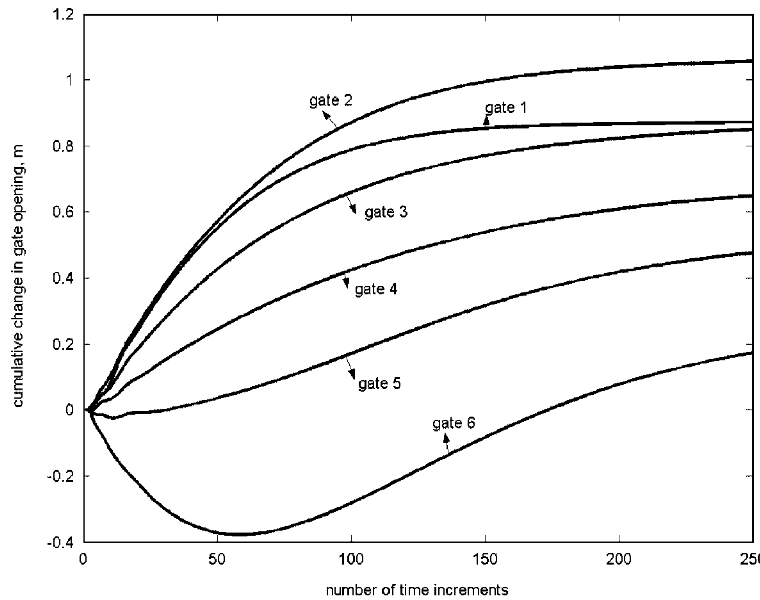


Figure 5. Cumulative gate openings.

state feedback, Kalman filter and fuzzy filter) and for each gate in the canal. The deviation in the gate openings for fuzzy filter was compared with the deviation in gate opening computed using optimal state feedback as well as steady-state Kalman filter. At the beginning, gate 1 had sharp peaks for all three design techniques. Since optimal state feedback has the best stability properties, the state feedback curves will be target loop. At gate 1, incremental gate openings for fuzzy filter were closer to optimal state feedback (target-loop function) than were those for the Kalman estimator. After 6000 s, gate 1 reached an equilibrium position for all three techniques. At gate 2, gate 3, gate 4, gate 5 and gate 6, the incremental gate openings for the Kalman filter were far away from the optimal state feedback in comparison to the fuzzy filter values. At the end of the simulation, the variations in the gate openings (for all gates) approached a constant value, indicating that a new equilibrium condition was established. Figure 5 demonstrates the cumulative gate openings of the irrigation canal. It was observed that gate 1, gate 2, and gate 6 had the highest changes in the gate openings. Also the final openings for gate 1 and gate 2 reached the highest values among the other gates (Figure 6). Since the variation in the gate openings were not a good indicator of the performance of a control algorithm [1], the variations in the flow depths at the downstream end of all the pools were computed. Figure 7 demonstrates the variations in flow depths for each pool. The variations in flow depths for the fuzzy filter were compared with the variations in flow depths computed using the optimal state feedback as well as the steady-state Kalman filter. Since pool 1 was the first pool of the irrigation canal, with an increase in flow rate into the lateral (turnout) or downstream demand, the depth of flow at the downstream end of pool 1 decreased rapidly and approached a maximum deviation of −0.17 m for the fuzzy filter, −0.165 m for the optimal state feedback and −0.15 m for the Kalman filter at approximately 2000 s from the beginning of the disturbance period. By the end of the simulation, the system returned

 

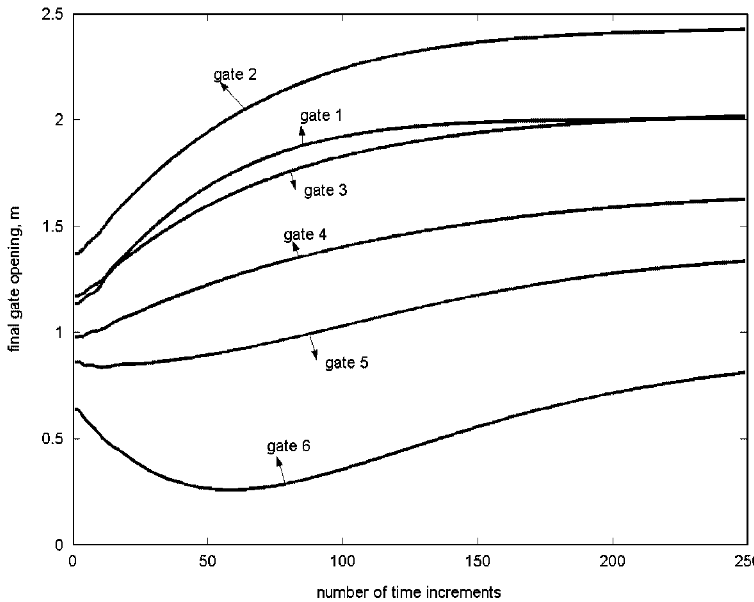


Figure 6. Final gate openings.

very close to the original equilibrium condition for all three techniques. In pool 2, first 1700 s of the simulation, the flow depth decreased dramatically and reached a maximum deviation of −0.145 m for the optimal state feedback, −0.143 m for the fuzzy filter and −0.125 m for the Kalman filter. The variations in flow depth in pool 3 reached −0.155 m for the fuzzy filter and −0.15 m for the optimal state feedback and −0.142 m for the Kalman filter around 1500 s of the simulation period. Pool 4 and pool 5 had less fluctuation in comparison to the other pools. Pool 6 had the highest variations in the flow depths and the flow depth at the downstream end decreased rapidly and approached a maximum deviation of −0.325 m for the optimal state feedback, −0.3 m for the fuzzy filter and −0.275 m for the Kalman filter at around 2000 s of the simulation period. The rapid decreases in the downstream depth of flow in each pool resulted in an attendant sudden increase in the gate openings at the upstream end of the each reach to release more water into the pool. However, because of the wave travel time, the depth of flow at the downstream end did not start to rise until around 1700 s. All the pools considered, the maximum deviation in depth of flow occurred at the first and last pools of the canal for all three design techniques. To meet the downstream target depth, the last pool had the highest fluctuations. The fluctuations in the first pool were because of releasing more water into the downstream pools and meeting the demand at the downstream end. It is obvious that the variations in flow depth for the fuzzy estimator are closer to the target-loop function (optimal state feedback) than were those for the Kalman estimator. In other words, Lyapunov-type fuzzy filter had better stability properties than Kalman filter in the control of irrigation canal. The demonstrated fuzzy filter not only guarantees the stability for the estimation but also leads to a precise estimation. Moreover, the fuzzy filter gain matrix can be easily computed by using the fuzzy rules.

 

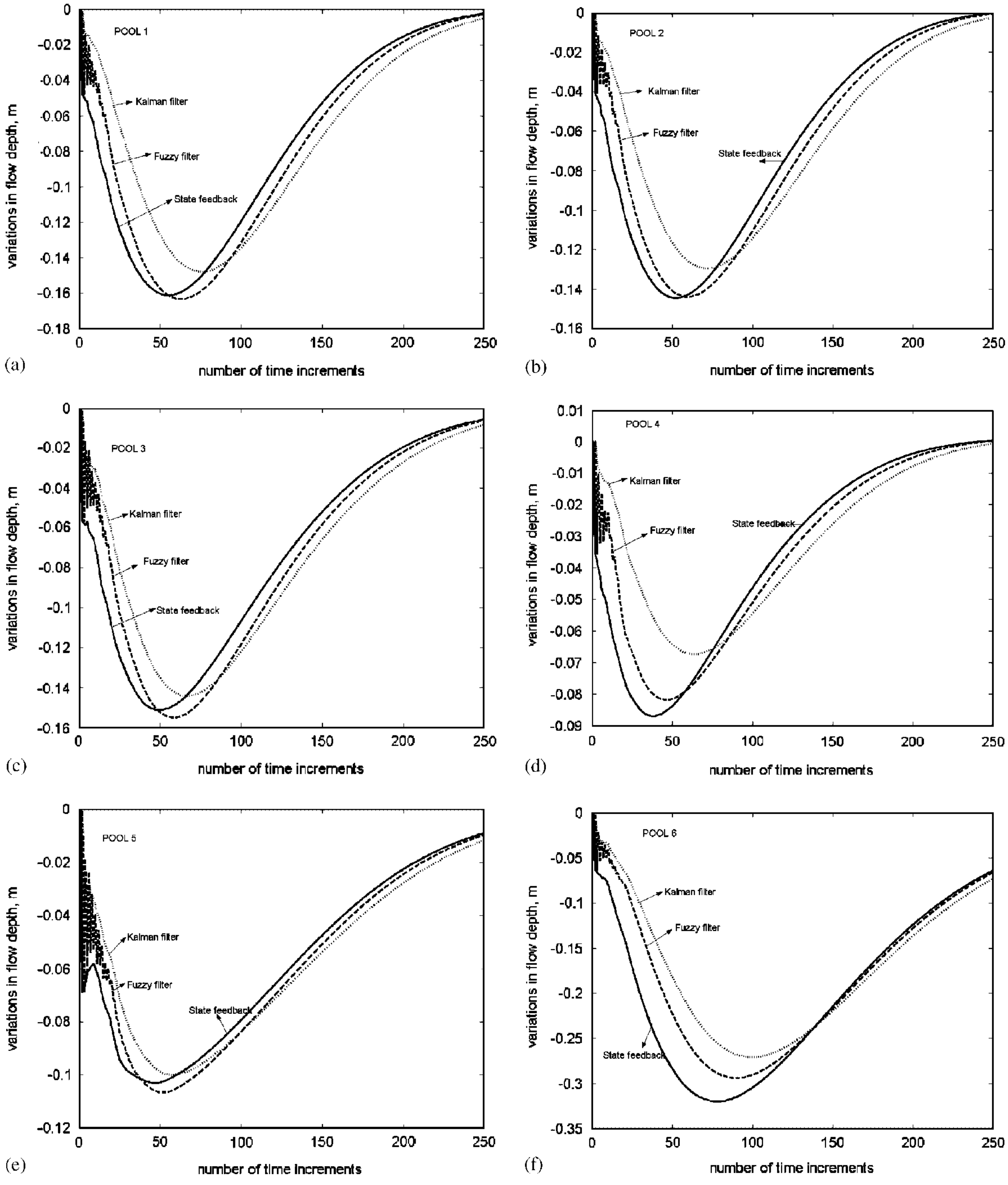


Figure 7. Variations in flow depths for optimal state feedback, Kalman filter and fuzzy filter.

## CONCLUSIONS

A fuzzy state estimator has been implemented in the control of multi-pool irrigation canals to estimate the state variables (flow depth and flow rate) at intermediate nodes based on the measured variables. The derivation of the fuzzy inference was based on Lyapunov function. The performance of the fuzzy estimator was compared with the performance of the optimal state feedback and Kalman filter in terms of variations in the depths of flow and the upstream gate openings. Since the optimal state feedback (assuming all state variables are measured) has the best robustness and stability properties, it is chosen as a target-loop function in this study. The results obtained from simulations indicate that the fuzzy estimator provides both good stability and performance in the control of irrigation canals. The advantages of fuzzy state estimator are its simplicity, effectiveness and stability. Overall, the performance of the fuzzy estimator technique for constant-level control was found to be better than the performance of the Kalman filter.

## REFERENCES

1. Reddy JM. Kalman filtering in the control of irrigation canal. *Applied Mathematical Modelling* 1995; **19**(4): 201–209.
2. Balogun OS, Hubbard M, DeVries JJ. Automatic control of canal flow using linear quadratic regulator theory. *Journal of Irrigation and Drainage Engineering* (ASCE) 1988; **114**(1):75–101.
3. Balogun OS. Design of real-time feedback control for canal systems using linear quadratic regulator theory. *Ph.D. Dissertation*, Department of Mechanical Engineering, University of California at Davis, U.S.A., 1985.
4. Reddy JM. Local optimal control of irrigation canals. *Journal of Irrigation and Drainage Engineering* (ASCE) 1990; **116**(5):616–631.
5. Reddy JM, Dia A, Oussou A. Design of control algorithm for operation of irrigation canals. *Journal of Irrigation and Drainage Engineering* (ASCE) 1992; **118**(6):852–867.
6. Malaterre PO. Modelisation, analysis and LQR optimal control of an irrigation canal. *Ph.D. Dissertation*, LAAS-CNRS-ENGREF-Cemagref, France, 1994.
7. Reddy JM, Raymond JG. Stochastic optimal and suboptimal control of irrigation canals. *Journal of Water Resources Planning and Management* (ASCE) 1999; **125**(6):369–378.
8. Corriga G, Sanna S, Usai G. Sub-optimal level control of open-channels. *Proceedings of the International ASME Conference*. ASME: New York, NY, 1982; 67–72.
9. Durdu OF. Estimation of state variables for controlled irrigation canals via a singular value based Kalman filter. *Fresenius Environmental Bulletin* 2004; **13**(11).
10. Durdu OF. Robust control of irrigation canals. *Ph.D. Dissertation*, Colorado State University, Fort Collins, CO, U.S.A., 2003.
11. Li TS, Lai CC. Lyapunov function base fuzzy state estimator. *IEEE ISCAS'95*, Seattle, WA, May 1995; 497–500.
12. Li T-HS. Estimation of one dimensional radar tracking via fuzzy-Kalman filter. *Proceedings of the IECON*, 1993; 2384–2388.
13. Lee CC. Fuzzy logic in control systems: fuzzy logic controller—Part I, II. *IEEE Transactions on Systems, Man and Cybernetics* 1990; **20**:404–435.
14. Kobayashi K, Cheok KC, Watanabe K, Munekata F. Accurate differential global positioning system via fuzzy logic Kalman filter sensor fusion technique. *IEEE Transactions on Industrial Electronics* 1998; **45**(3): 510–518.
15. Zimmermann HH. *Fuzzy Sets Theory and its Applications*. Kulwer-Nihjoff Publishing: Boston, MA, 1995.
16. Chaudhry MH. *Open-channel Flow*. Prentice-Hall: Englewood Cliffs, NJ, 1993.
17. Kwakernaak H, Sivan R. *Linear Optimal Control Systems*. Wiley: New York, 1972.
18. Garcia A, Hubbard M, De Vries JJ. Open channel transient flow control by discrete time LQR methods. *Automatica* 1992; **28**(2):255–264.
19. Sergio M. *Hydraulics of Open Channel Flow*. ASCE Press: Reston, VA, 1998.
20. Malaterre PO. Multivariable predictive control of irrigation canals. Design and evaluation on a 2-pool model. *International Workshop on Regulation of Irrigation Canals*, Morocco, 1997; 230–238.
21. Reddy JM. Simulation of feedback controlled irrigation canals. *Proceedings USCID Workshop*, *Modernization of Irrigation Water Delivery Systems*, 1999; 605–617.

 

22. Kailath T. *Linear Systems*. Prentice-Hall: Englewood Cliffs, NJ, 1980.
23. Tewari A. *Modern Control Design with Matlab and Simulink*. Wiley: New York, 2002.
24. Durdu OF. Regulation of irrigation canals using a two-stage linear quadratic reliable control. *Turkish Journal of Engineering and Environmental Sciences* 2004; **28**:111–120.
25. Pai NS, Li TS. State estimation via Lyapunov-type fuzzy filter. *Electronic Letters* 1999; **35**(12):1028–1030.
26. Hubbard M, De Vries JJ, Balogun OS. Feedback control of open-channel flow with guaranteed stability. *Proceedings of the 22nd Congress of IAHR*, Lausanne, 1987.

 